LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2025**

UMT2MC02 – INTEGRAL CALCULUS

Date: 29-04-2025 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 PM

Q.No	Levels	SECTION A - K1& K2 (CO1)
		Answer ALL the Questions **(10 x 2 = 20)**
1	K1	Define Gamma function.
2		Show that $\beta(m,n)=\beta(n,m)$.
3		Evaluate $\int_0^1\int_0^2 xydxdy$.
4		Write the directional derivative of φ at the point $P(x,y,z)$ in the direction cosines l,m,n.
5		What is known as Quadrature?
6	K2	Find $\nabla\varphi$ for the scalar point function $\varphi(x,y,z)=xyz$ at the point (2,1,1).
7		Write the formula for work done by a force.
8		Write the formula to evaluate surface integral, whose projection on xoy plane.
9		State Green's theorem.
10		State Gauss divergence theorem.
		SECTION B– K3& K4 (CO2)
		Answer ALL the Questions **(4 x 10 = 40)**
11	K3	Prove that (i) $\Gamma(n+1)=n!$, where n is a positive integer. (ii) $\int_0^{\pi/2} sin^7\theta cos^5\theta d\theta=\frac{1}{120}$.
		[OR]
12		Find the area of the cardioid $r=a(1-cos\theta)$..
13		Evaluate $\iint xydxdy$ over the region bounded by the lines $x=0;\ y=0$ and $x+y=1$.
		[OR]
14		Prove that $\vec{A}=(2x+yz)\hat{\imath}+(4y+xz)\hat{\jmath}-(6z-xy)\hat{k}$ is solenoidal as well as irrotational and also find the scalar potential of $\vec{A}$.
15	K4	(i) If $\vec{F}=x^2y\hat{\imath}+y^2z\hat{\jmath}+z^2x\hat{k}$, then find $curl\ curl\ \vec{F}$. (ii) If $\vec{A}$ is a vector point function, then prove that $\nabla.(\nabla\times\vec{A})=0$.
		[OR]
16		If $\vec{F}=3xy\hat{\imath}-y^3\hat{\jmath}$, compute $\int_C \vec{F}.d\vec{r}$ along $y=2x^2$ from (0,0) to (1,2).
17		Evaluate $\iint_S \vec{F}.\hat{n}ds$, where $\vec{F}=z\hat{\imath}+x\hat{\jmath}-y^2z\hat{k}$ and S is the surface of the cylinder $x^2+y^2=1$ include in the first octant between the planes $z=0$ and $z=2$.
		[OR]
18		Using Green's theorem show that $\int_C (3x^2-8y^2)dx+(4y-6xy)dy=20$, where C is the boundary of the rectangular area enclosed by the lines $x=0, x=1, y=0, y=2$ in the xoy plane.

SECTION C – K5 & K6 (CO3)		
	Answer ALL the Questions	**(2 x 20 = 40)**
19	K5	(a) Derive the relation between Beta and Gamma functions. (b) Express $\int_0^1 x^m(1-x^n)^p\,dx$ in terms of Gamma functions and evaluate the integral $\int_0^1 x^5(1-x^3)^{10}\,dx$. **(10+10)** [OR]
20		(a) Find the area enclosed between an arc of the cycloid $x = a(\theta - sin\theta), y = a(1 - cos\theta)$ and its base. (b) Evaluate $\iiint(x^2 + y^2 + z^2)dxdydz$ taken over the volume enclosed by the sphere $x^2 + y^2 + z^2 = 1$. **(10+10)**
21	K6	(a) Prove that $\nabla \times (\nabla \times \vec{F}) = \nabla(\nabla.\vec{F}) - \nabla^2\vec{F}$. (b) Evaluate $\iiint_V (\nabla.\vec{F})\,dV$, where V is the region bounded by $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4$. **(10+10)** [OR]
22		Verify Gauss divergence theorem for $\vec{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$ over the cube bounded by $x = 0, x = 1;\ y = 0, y = 1;\ z = 0, z = 1$.

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